Navigation in Mathematical Documents

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Abstract. Mathematical documents are not only hard but fun to write,

they are equally hard but fun to read. Unfortunately, the \fun" part

comes only in, when one has mastered the art of using mathematical

vernacular and grasped all relevant context dimensions. Even though

this seems an immanent issue for mathematical authors only, it is also

for mathematical readers. In this paper, we argue for the need for more

reader assistance in mathematical documents. In particular, we believe

navigation to be a top candidate in this regard. Thus, we elaborate design

opportunities for (semantically) helping the reader to navigate, which we

anchor around the PlanetMath website.

1 Introduction

In recent years, the traditional notion of \document" changed drastically from

a \_nished product accessible only to a selected group of people to a potentially

open access, widely distributable, collaborative and exible artifact. Documents

are considered modern if they are active documents, i.e., documents that use

a presentation engine to (re- or inter-)actively adapt the surface structure of

the document to the environment or user input. Spreadsheets are the paradig-

matic example for such an active document type: If a spreadsheet author adds

a column to a spreadsheet, all cell references in underlying formulas are au-

tomatically updated by the presentation engine. Active documents exploit the

distinction between form and content (like cell layout vs. computed cell values

in spreadsheets).

Information on the Web is also often encoded in active documents (e.g.

HTML- or XML-\_les). But Web documents frequently carry yet another prop-

erty which non-Web documents don't have: The linear communication structures

of traditional documents are replaced by multidimensional, multifunctional and

multimedial hypertext structures, in which information is distributed according

to the \Net Communication Model" (see e.g. [Flu96] or Sect. 2.1).

The central idea of this paper is the observation, that mathematical docu-

ments rely heavily on the Net Communication Model, which makes them hard

to read without supporting services. Thus, we want to suggest potential reading

services for modern math documents.1 We \_rst point out the underlying Net

1 Please note that this is a workshop paper aiming at discussions around this topic.

Thus, we do not consider the ideas fully worked out yet.

Communication Model in mathematical documents (Sect. 2). In order to read

(and understand) a mathematical document, this either requires a highly ac-

tive reader's mind or active documents that o\_er elaborate reader assistance. In

Sect. 3 we look at an exemplary such service, which draws on the Net Commu-

nication Model in a speci\_c category of active math documents: Semantic navi-

gation in spreadsheets. Finally, we envision more navigational features in active

mathematical documents like articles on the PlanetMath website2 in Sect. 4.

2 The Net Communication Model in Math Documents

The idiosyncracies of mathematical vernacular have often been discussed. Many

researchers started out hoping to \_nd a nice model for it, so that the process

of either reifying a human's mathematical knowledge into reviewable documents

(\codi\_cation") or converting existing math documents into formal documents

(\formalization") could be supported or even automated. The same kind of prob-

lem arises when math educators teach math: young mathematicians have to learn

\. . . becoming conversant in the language of mathematical discussion." [Day11].

Mathematical vernacular turns a math document into an intellectual arti-

fact, i.e. according to [dS05, p. 10],

{ \it encodes a particular understanding or interpretation of a problem situation

{ it also encodes a particular set of solutions for the perceived problem situation

{ the encoding of both the problem situation and the corresponding solutions is fundamentally

linguistic (i.e., based on a system of symbols [. . . ]); and

{ the artifact's ultimate purpose can only be completely achieved by its users if they

can formulate it within the linguistic system in which the artifact is encoded"

In particular, de Souza points out that intellectual artifacts require that

producer and consumer use the same language. Math documents as intellecutal

artifacts make use of a linguistic encoding system. As mathematical vernacular

itself does not seem to be too systematic (otherwise all the research already

done would have resulted in \_nding this system), it only seems to support the

encoding system.

Therefore, in this section, we take a look at the underlying discourse structure

of mathematical vernacular and \_nd an underlying Net Communication Model.

2.1 The Net Communication Model (NCM)

Media-theorist Vilem Flusser suggested several discourse models for commu-

nication in [Flu96]. He distinguishes four communication discourse types:

Pyramid The pyramid discourse preserves information very well, as it is

centered around getting con\_rmation about every obtained and possibly re-

coded information by its sender. Edited articles can serve as an example for

this discourse type, as the information is checked by the author in the \_nal

proof copy before it is published.

2 www.planetmath.org, relaunched shortly based on Planetary services (see [?])

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Theatre The theatre discourse is characterized by a sender and a receiver ex-

changing information, protected against communication noise and thus mis-

understanding by a concave theater screen. As long as the actors constrain to

the rule to be within the protection area of the screen while communicating,

the information entropy is very low. A document handed from the author

to colleague in his \_eld \_ts this description. The safeguarding screen here is

the resp. Community of Practice (CoP) (see [LW91]).

Graph The tree or rather graph discourse, Flusser describes as sciences'

discourse. The original sender (=author) creates information to be commu-

nicated, this information is broken down into information fragments, that

in turn are picked up, re-coded and further communicated. Flusser calls it

\centrifugal information distribution" and points to its inherent information

creation while not conserving the information's original content. In many

related work sections in scienti\_c documents we can \_nd such distributed

information fragments, that give the setting for innovative ideas, i.e., new

information.

Amphitheatre The \_nal discourse type, Flusser presents is the amphithe-

atre discourse. Here, the information is neither send by one sender (but by

many) nor is there a screen available against which the communication hap-

pens. This is the modern, web discourse type, the Net Communication

Model (NCM). The information entropy is extremely high, but informa-

tion is communicated extremely fast and broadly.

2.2 The NCM in Math Documents

Now we want to look at the discourse type of math documents.

{ Pyramid: As it is very important for mathematicians to conserve the \ob-

jectivity" and \truth" of statements, this safest-of-all discourse type seems

highly attractive for mathematical communication in documents. Publica-

tion in a journal is still the most honorable publication format for mathe-

maticians, and here the author indeed carefully checks its proof before it is

published. But this is only a check of the form. The real question concerns

a document's content, that is, whether used information fragments by other

authors are checked by those authors? Here, the answer is no. In particular,

the pyramid discourse type seems not to \_t math documents.

{ Theatre: The \objectivity" and \truth" in math documents are kept by a

more or less rigid reviewing system. This is organized within the concerned

Community of Practice, which serves as a theatre screen for the safety of the

communication of mathematical information. Therefore, we conclude at \_rst

glance that mathematicians prefer to practice this communication discourse

type. Unfortunately, this type does not support information creation, but

mathematicians love the creativeness in math, for example:

\It is somewhat remarkable that a subject with such high and objective

standards of logical correctness should at the same time provide such

opportunity for the expression of playful genius. This is what attracts

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many people to the study of mathematics, particularly those of us who

have made it our life's work." [Day11]

So the theatre discourse type doesn't seem to \_t too well to math documents.

{ Graph: But the graph discourse type supports creativeness, so we can con-

sider this a \_tting type for math documents? One of the most successful tools

of mathematicians is their practice of aligning di\_erent discourses according

to their purpose, for instance:

\Mathematics is about ideas. In particular it is about the way that

di\_erent ideas relate to each other." [Ste90, p. 2]

Moreover, modularization is another key mathematical practice, which en-

ables \_ne-tuned aligning of theories. We conclude, that mathematicians use

the graph discourse type in their documents.

{ Amphitheatre: But how do mathematicians preserve \objectivity" and

\truth" in their documents at the same time? They screen the information

by their assumption of a math document to be self-contained. In particular,

any reader can persuade herself of the truthfulness of inferences/conclusions.

Therefore, even if a miscommunication happened in the discourse at some

point, the derived statements are true in their local context. Stewart gave

an example of the vernacularous depth of mathematics by letting a \_ctious

mathematician (in a discussion with a \_ctional layman) exclaim

\You can't get a feeling for what's going on without understanding the

technical details. How can I talk about manifolds without mentioning

that the theorems only work if the manifolds are \_nite-dimensional

para-compact Hausdor\_ with empty boundary?" [Enz99, p. 45]

This example demonstrates that the self-containedness has its natural limits.

Those links outside math documents are part of the amphitheatre discourse

type.

In summary, math documents are built on basis of the amphitheatre discourse

type, but including parts of the theatre and the graph discourse types locally.

In particular, they yield the Net Communication Model with local creativity-

enabling provenances and local safe-guards.

3 Math Reader Assistance:

Semantic Document Navigation

The di\_culty of reading math documents becomes clear directly: To understand

the locally used theatre screens, the reader has to be either part of the resp.

Community of Practice or has to has access to its knowledge in all of its dimen-

sions (see e.g. [KKL10]). Moreover, math documents frequently contain holes

that refer to the implicit knowledge of the resp. CoP, which makes that speci\_c

part in the document potentially ambiguous based on the graph discourse type.

This means that readers may \_ll in the missing parts incorrectly or not at all.

Therefore, we argue for more reader assistance in mathematical documents in

general.

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3.1 Navigation

The showcased issues centered around a reader's orientation problems, thus, we

consider navigation support a central service for these readers. In [IM00] navi-

gation was introduced as a service that allows users \to browse a huge document

[. . . ] without problems". Originally, especially in the Nautics discipline, it de-

scribes the \process of monitoring and controlling the movement of a craft or

vehicle from one place to another" [Wik11]. When transferred to the \sea of

information"(e.g. [McI03]) available on the Web, navigation becomes the (elab-

orate) process of steering towards looked-for information goals.

Dourish and Chalmers introduced in 1994 the term \semantic naviga-

tion". In particular they stated that the

spatial organisation of data has been a highly visible component of a number of

information systems.[. . . ] Indeed, it's the notion of spatial arrangements which

encourages (and legitimises) the notion of navigation of information systems.

However, in navigation (as opposed to organisation), this use of the \spatial"

is a convenient gloss for a di\_erent organisation, which we refer to here as

\semantic".[DC94]

They realized that in hypertext systems the primary form of navigation is of a

semantic nature, in particular, depending on the semantic properties of existing

links. Going beyond one-dimensional hyperlinks we therefore like to transfer this

well-known notion of navigation to non-Web, active documents.

3.2 Semantic Document Navigation in SACHS

A possible way to provide reader assistance was presented for spreadsheets as

active, mathematical documents in [KK11]. Even though they do not contain

the usual mathematical vernacular, they also operate in the same discourse type

fashion. Their local screens are built for the data of the document, their local

provenance as well. Underlying formulae and charts are computed resp. drawn

on the spot, so that the presented data are locally \true" data. The reader's

interpretation of the data depend on the locally given, provenanced-dependent

context information like headers. The known usability problems wrt. to spread-

sheets were analyzed to be either false formulae, false data interpretation or

input data errors (e.g. [KK10]).

We realized with the SACHS system [KK11] a semantic help system for spread-

sheet documents, which includes interesting document navigation features, that

we summarize next.

Let us start with introducing our running example document (see Fig. 1):

An MS Excel spreadsheet based on [Win06], which can be considered a simple

controlling system. It shows a pro\_t-loss statistics over a time period, including

cost and revenue data. The purpose and meaning of the spreadsheet seem clear

enough, but as soon as we ask

{ for which company this is a controlling system, or

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Fig. 1. A Simple Controlling System Using MS Excel after [Win06]

{ whether the numbers are given in millions of Dollars or Yen, or

{ what the is de\_nition of \projected data"

we realize right away that even in such a simple document the explicit knowledge

is just the tip of the iceberg of the (background) knowledge necessary to interpret

it. The SACHS system was designed to draw on a semi-formal formalization of this

background knowledge as domain ontology.

The spreadsheet objects that carry meaning are the cells. They are inter-

preted by the user both wrt. the grid layout (like within a table with an assigned

row and column speci\_cation) and via the underlying formula. With SACHS we

o\_er a third dimension of interpretation by providing access to the background

knowledge based on the alignment of cells with concepts in a domain ontology. In

particular, we realized an embedded user assistance method by using cell clicks

as entry points for the help system, that is, every click on a cell generates help.

Here, we are only interested in the help features \Dependency Graph" and

\Search" box, since these two allow the user semantic document navigation.

The Dependency Graph

Let us \_rst look at what happens when the user clicks on cell [B15] (Total

Expenses, 1984). Then a new window (as seen in Fig. 2) is opened displaying at

the top the concept connected to the selected cell. All concepts, which this top

concept directly depends on, are shown on the second level.

If the user wants to elaborate on a speci\_c concept like \Utility Costs", then

a click on the corresponding node expands it by another level. The user is free to

drill down into ever more abstract information available in the domain ontology;

see Fig. 3 for an example path.

In a nutshell, the dependency graph enables the user to explore the back-

ground knowledge according to her own mental map of the concerned knowledge,

her experience with it, her situation-dependent interests and time-frames. Cells

are reinterpreted as hyperlinks to a domain ontology and moreover, the nodes in

the dependency graphs are themselves links to further concepts in this ontology.

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Fig. 2. Dependency Graph for Cell [B15]: Level 1 and 2

Iske [Isk02] distinguishes three navigational options based on a Network

Communication Model: un- and directed browsing, and guided tours. A spread-

sheet reader can explore the document and its previously hidden knowledge on

her own, therefore SACHS enables undirected browsing as navigational service.

Moreover, it o\_ers directed browsing features as it is designed around the central

semantic object for the interpretation of data within a spreadsheet | the cell

(see [KK09]). The author of the spreadsheet and presumably of the resp. back-

ground knowledge governs the form of the dependency graph, thus the graph

feature in SACHS can be also considered to promote paths or guided tours as a

navigation option.

One can argue that this kind of navigation is mostly happening in the \doc-

ument behind the original document": the background knowledge captured in a

domain ontology. But the navigation functionality was also extended to cross-

modality navigation by taking the alignment of concepts and cells as semantic

document navigation cues.

In SACHS we mashed-up the graph-based interface with spreadsheet focus

operations to enable spreadsheet navigation via the de\_nitional structure of

the intended functions in the structured background ontology. Note that the

nodes e.g. in Fig. 3 have distinct colors. This color-coding indicates whether the

concept in a node is connected to a speci\_c cell in the workbook: An aligned

concept node in the dependency graph carries a hyperlink to the resp. cell.

Let us look at Fig. 3 for example. There are two nodes in darker grey: \Actual

Expenses at SemAnteX" and \Actual Utility Costs at SemAnteX". Clicking the

node \Actual Utility Costs at SemAnteX" moves the spreadsheet focus to cell

[B10]. Then the user can switch back to the original position in the spreadsheet

by clicking the top node, here the \Actual Expenses at SemAnteX" node.

This way a user can get a good orientation on how the spreadsheet works

and an overview over the various dependencies between cells.

Searching in the Background Knowledge

The search feature enables the user to reach through to the information available

in background knowledge.

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Fig. 3. Dependency Graph for Cell [B15]: More Levels

Concretely, users can type into the search box a string of characters which is

used to search all concept titles for this string.

Once an element in the search result is selected by a user, the according

explanation is presented in this area. For instance, she likes to know more about

the concept \Steady State Prognosis", so she clicks on the according item in the

list and its de\_nition is shown (see Fig. 4). The concept \Actual Salary Costs per

Time Interval at SemAnteX" is aligned to the cell [B9] in our running example.

A click on the \Select Cell!" button would thus result in a navigation to this

aligned cell.

On the lower right hand side of the \Search Results" area we can also \_nd the

\N!" command button. Pushing this results, on the one hand, in the selection

of the next aligned concept in the search list, and on the other hand, in the

navigation to the resp. cell in the according spreadsheet. The order is determined

by the current cursor position within the list downwards and on user request

starting on its top again. This feature enables the user who is unfamiliar with

the concepts generally or concept titles particularly to judge by herself whether

the shown concept is the one she was looking for to begin with.

In summary, we observe that both the concept graph as well as the hit list (the

list of search results) can be used by the user as \navigation panels". The former

is arguably more semantic (as it uses the conceptual dependency structure),

whereas the latter is more mnemonic (it goes via the concept names). Their

utility is largely due to the fact that they allow the user to focus on a particular

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Fig. 4. Explanation for \Actual Salary Costs at SemAnteX" from Background Knowl-

edge

aspect | the dependency cone of concepts leading up to the value of a particular

cell or the concepts mentioning a particular string | and use these as a jump list

for complex document collections. Indeed, commentators on the SACHS system

always highlighted the added-value of being able to disregard worksheet and

potentially even workbook limitations when navigating.

4 Math Reader Assistance of the Future:

More Navigation

In order to envision more helpful navigation features, we imagine them for a

web-based collection of documents as gathered on the PlanetMath website [?].

4.1 Articles Graph

Fig. 5. Articles Graph

Consider for example the short article http://

planetmath.org/PrimeAn1.html. The blue strings

indicate links to other PlanetMath articles. The rep-

resentation of the dependencies of this article on

the other articles as dependency graph on the side,

would give a clear indication of what this article is

about. In contrast to the semantic document navi-

gation in SACHS, the development of the graph level

by level doesn't seem sensible as the user can do this by her own by using the

hyperlink structure itself. But using this given hyperlink structure the reader

quickly looses the overview and maybe doesn't even remember where she started

at (think of the \magical number 7" in interaction design rules). A graph would

be very helpful, in particular when deciding that a followed branch wasn't worth

the e\_ort.

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4.2 Bibliographic Graph

Fig. 6. Bibliographic Graph

Whenever one sees a list of biblio-

graphic references, one is wondering

which of all are the important ones.

A hint could be delivered if the bibli-

ographic references were presented in

a \bibliographic graph", a graph con-

taining all the references of an arti-

cle and all recursively discovered ref-

erences for each reference. The impor-

tant ones are the likeliest to be ref-

erenced from the present references

themselves (somewhat in analogy to Google's pagerank algorithm). If other ar-

ticles referring to some of the present references, then this information may also

be included into a graph. Naturally, this information could also be used to rank

the references of an article.

4.3 Formula Graph

Fig. 7. Formula Graph

Other objects that appear fre-

quently in math documents

are formulae. If we take these

serious as semantic objects,

then a graph visualization also

seems sensible. With the recent

progress in math search facili-

ties, structurally equivalent for-

mulae can be found in other ar-

ticles. Moreover, subformulae used in other articles can be discovered as well. If

an article uses the same formula as the original article and additional ones, then

these can also be used as a hyperlink in a respective graph.

5 Conclusion

In this paper we address the issue of the Net Communication Model together with

some math speci\_c extensions as a discourse model for math documents. This

speci\_c model makes it on the one hand hard to write and read math documents,

but on the other hand it also provides the means to create a document that

can only be understood from a holistic point of view. Once the author or the

reader have been enabled to enjoy this view, fun is also part of the consuming

process for that document. The holistic view depends on several dimensions

of the underlying Net Communication Model. A math document can only be

created if the author has mastered these dimensions, but not every reader has

done so. Here, navigational features appear naturally as help services, as they

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serve as access points to multi-dimensional information. We gave an example for

semantic document navigation in spreadsheet documents via the SACHS system

and envisioned some more navigational design opportunities for a collection of

active, web-based math documents in the PlanetMath system.

We believe that such reader assistance will be particularly useful and gives

them new, and e\_cient access to salient parts of math documents. In turn, this

will induce a better overview and a deeper understanding of the concepts in users

| and for some even enable an enjoyable reading experience of math documents.

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